

Continuous quantum measurement in spin environments

Dong Xie* and An Min Wang†

Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui, China.

We derive a formalism of stochastic master equations (SME) which describes the decoherence dynamics of a system in spin environments conditioned on the measurement record. Markovian and non-Markovian nature of environment can be revealed by a spectroscopy method based on weak quantum measurement (weak spectroscopy). On account of that correlated environments can lead to a nonlocal open system which exhibits strong non-Markovian effects although the local dynamics are Markovian, the spectroscopy method can be used to demonstrate that there is correlation between two environments.

PACS numbers: 42.50.Dv, 42.50.Pq, 03.65.Yz

I. INTRODUCTION

Generalized or weak quantum measurement becomes more and more important in a lot of fields such as quantum feedback control [1, 2], quantum metrology [1], quantum information [3-5], and the study of quantum-classical transitions [6, 7]. The existing theories consider continuous weak measurement of simple open quantum systems with Born-Markov decoherence models [5, 8-10]. However, there is a lack of theoretical framework to extend the exceptional capacities of weak measurement method for system in non-Markovian decoherence environments. Recently, Shabani et.al. [11] addressed the demand in a bosonic environment. Therefore, they left an interesting open question: spin environments present more significant challenges for this analysis [12]. Following that, we analyze the decoherence dynamics of a system in spin environments.

We further develop a cavity quantum electrodynamics theory for continuous measurement of an arbitrary quantum system coupled to a spin environment. In this framework, we derive SME that describes the conditional evolution of system in the presence of Markovian and non-Markovian decoherence effects. As in the case of photocurrents, it is often convenient to characterize the dynamics by the spectrum of the current. By using the Itô rules, we can numerically calculate the reduced two-time correlation function [1] and draw the corresponding spectrograms, which can reveal Markovian and non-Markovian nature of decoherence dynamics in spin environments.

We also find an application of the spectroscopy techniques. Lain et.al [13] demonstrated that enlarging an open system can change the dynamics from Markovian to non-Markovian. Therefore, we derive SME about two quantum systems and show that correlated environments can lead to a nonlocal open system dynamics which exhibits strong non-Markovian effects although the local dynamics is Markovian by the spectroscopy techniques.

The rest of paper is arranged as follows. In section II, we describe the model and derive SME in a spin environment. Markovian and non-Markovian nature of the decoherence dynamics in spin environments are explored in section III. In section IV, we discuss about an application of the spectroscopy techniques to show non-local non-Markovian effect. We deliver a conclusion and outlook in section V.

II. MODEL OF A SYSTEM-CAVITY

Let us consider that a quantum system couples with a single cavity mode. The total Hamiltonian of the cavity and the system is given by ($\hbar = 1$)

$$H_{SC} = H_S + \omega_c a^\dagger a + \hat{\lambda}(a^\dagger + a), \quad (1)$$

where H_S , $\omega_c a^\dagger a$ denote the Hamiltonian of the system and the cavity, respectively; the last term represents a system-cavity coupling Hamiltonian H_{int} . We consider the dispersive regime where the cavity is relatively far off detuned from the system resonance frequencies, i.e., for $|\hat{\lambda}_{jk}| = |\langle j|\hat{\lambda}|k\rangle| \ll |\omega_c - (\Omega_k - \Omega_j)|$ with the spectral decomposition $H_S = \sum_j \Omega_j |j\rangle\langle j|$. Then, by applying a generalized dispersive transformation $U_D = \exp[Xa^\dagger + X^\dagger a]$ and the rotating wave approximation and neglecting two photon creation and annihilation processes, the total Hamiltonian can be turned into the desired form [1, 14]

$$H_{SC}^D = U_D H_{SC} U_D^\dagger \approx H_S^D + \omega_c a^\dagger a + O_S a^\dagger a, \quad (2)$$

where $X = \sum_{jk} \frac{\lambda_{jk}}{\omega_c - (\Omega_k - \Omega_j)} |j\rangle\langle k|$, modified system Hamiltonian $H_S^D = H_S - \frac{1}{2}(X^\dagger \hat{\lambda} + \hat{\lambda} X)$, and the system operator $O_S = \frac{1}{2}[\hat{\lambda}, X^\dagger - X]$. The system operator O_S can adjust the cavity frequency, and therefore measuring the phase of the leaking photons reveals information about the system.

Next, we consider the cavity is driven by a classical light with a single frequency component, i.e. $\xi(t) = \xi_p e^{-i\omega_p t}$, where ω_p is close to ω_c . The corresponding dispersive Hamiltonian for a classical drive is

$$H_{drive}^D = \xi(t) U_D a^\dagger U_D^\dagger + h.c. \approx \xi_p e^{-i\omega_p t} a^\dagger (1 + \Lambda) + h.c., (3)$$

*Electronic address: xiedong@mail.ustc.edu.cn

†Electronic address: anmwang@ustc.edu.cn

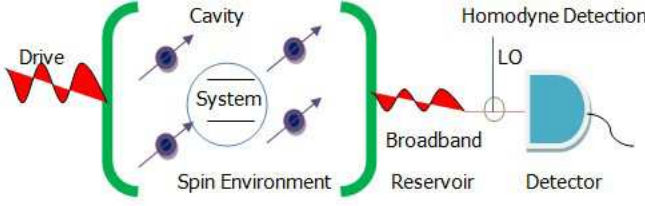


FIG. 1: A quantum system that is coupled to a spin environment is probed by a single mode cavity resonator coupled to a spin environment. By the way of homodyne detection, a detector continuously measures the photons leaking out of the cavity to obtain information about the system dynamics.

where $\Lambda = \frac{1}{2}[X^\dagger, X]$.

The dynamics of the system and cavity is further influenced by two sources of ambient interactions: a broadband (Markovian) reservoir R that couples to the cavity, causing photon leakage, and an environment E that induces decoherence via its coupling to the system. Here, the photon leakage process is modeled by a reservoir of electromagnetic modes, $H_R = \sum_r \omega_r b_r^\dagger b_r$, which is linearly coupled to the cavity mode: $H_{CR} = \sum_r g'_r (b_r + b_r^\dagger) \hat{R}$, where b_r denotes the lowering operator of the r th mode at frequency ω , \hat{R} is some Hermitian electromagnetic reservoir operator (we choose a special form $\hat{R} = a + a^\dagger$ in the following section), and g'_r represents coupling strengths. The effect of such reservoir can be captured by the Drude-Lorentz form spectral density, $J(\omega) = 2\mu\nu \frac{\omega}{\omega^2 + \nu^2}$ with coupling strength μ and cut-off frequency ν . The effect of the environment E is then treated as a sum of local baths of two level spins with Hamiltonian $H_E = \sum_{k=1}^N \omega_k \sigma_Z^k$, where σ_Z^k is the k th Pauli spin operator. The system-environment coupling is of the form $H_{SE} = \sum_{k=1}^N g_k \sigma_Z^k \hat{S}$ with some system operators \hat{S} and coupling strength g_k .

Now, in the dispersive frame, the Hamiltonian is

$$H_{CR}^D = \sum_r g'_r (b_r + b_r^\dagger) [(a + a^\dagger)(1 + \Lambda) - (X + X^\dagger)], \quad (4)$$

$$H_{SE}^D = \sum_k g_k \sigma_Z^k (\tilde{S} + Q a^\dagger a + G), \quad (5)$$

where $\tilde{S} = \hat{S} - 1/2\{X^\dagger X, \hat{S}\} + X^\dagger \hat{S} X$, $Q = (D[X] + D[X^\dagger])\hat{S}$, and $G = -[X^\dagger, \hat{S}]a + [X, \hat{S}]a^\dagger$. The operator \tilde{S} represents the effective system and environment coupling. The equations for the system-cavity-environment dynamics is obtained by Born-Markov approximation

$$\frac{d\rho_{SCE}}{dt} = (\mathcal{L}_{SCE} + \mathcal{L}_{leak})\rho_{SCE}, \quad (6)$$

with $\mathcal{L}_{SCE}\rho_{SCE} = -i[H_{SC}^D + H_{SE}^D, \rho_{SCE}] + \kappa D[X]\rho_{SCE}$ and the cavity leakage rate κ determined by $J(\omega)$. $\kappa D[X]$ is the Purcell type of system decoherence modification and is also a part of the measurement backaction. The

superoperator $\mathcal{L}_{leak} = \kappa D[a(1 + \Lambda)]$ denotes the modified cavity leakage process.

Like the way in ref. [11], we arrive at the following SME to describe the homodyne measurement of the system, cavity and environment [1]

$$d\rho_{SCE} = (\mathcal{L}_{SCE} + \mathcal{L}_{leak})\rho_{SCE}dt + \sqrt{2\eta\kappa}\mathcal{H}[a(1 + \Lambda)e^{-i\phi}]\rho_{SCE}dW, \quad (7)$$

where η denotes the efficiency of a detector and the infinitesimal increment dW represents a Wiener process [1]. The corresponding detector current $I(t) = \frac{dQ}{dt}$ can be written as

$$dQ = 2\eta\kappa((1 + \Lambda)(ae^{-i\phi} + a^\dagger)e^{i\phi})dt + \sqrt{2\eta\kappa}dW. \quad (8)$$

The bad cavity regime [15, 16] is considered for ensuring that the detection information reflects only the quantum state of the system. A good criterion for applicability of the bad cavity parameter regime is given by [11]

$$\kappa \gg \|O_S\|_1 (1 + |\alpha|^2), \quad (9)$$

with the bare cavity coherent state $|\alpha = \xi_p/i\kappa\rangle$ for $\omega_c = \omega_p$.

Then, for a relatively high leakage (low finesse) cavity, we use the standard approach as described in Refs. [11, 15, 16].

1- Write Eqs. (7,8) in the frame rotating with the drive frequency ω_p .

2- Project the cavity to the ground state by the transformation $\rho_c \rightarrow \exp(\alpha a^\dagger - \alpha^* a)\rho_c D(\alpha) \exp(-\alpha a^\dagger + \alpha^* a)$.

3- Represent the system-cavity-environment density matrix as $\rho_{SCR} = \sum_{lk} (\rho_{SCR})_{lk} |l\rangle\langle k|$, where $|k\rangle$ is the cavity k photon state in the displaced framework, and $(\rho_{SCR})_{lk}$ is the corresponding system operator. Expand the density matrix ρ_{SCR} to the second order of the perturbative parameters $\varepsilon = \frac{1}{\kappa}\{(|O_S| + \Delta)(1 + |\alpha|^2)\}$. The high leakage condition corresponds then to $\varepsilon \ll 1$.

Following the above procedure, we can obtain the reduced SME

$$d\rho_{SE} = \mathcal{L}_{SE}\rho_{SE}dt + \{-i(\xi_p\alpha^* + \alpha^*\xi_p)[\Lambda, \rho_{SE}] - i|\alpha|^2[O_S, \rho_{SE}] + \frac{\kappa|\alpha|^2}{\kappa^2 + \Delta^2}D[O_S]\rho_{SE} + \frac{i\Delta|\alpha|^2}{\kappa^2 + \Delta^2}[O_S^2, \rho_{SE}]\}dt + \sqrt{2\eta\kappa}\mathcal{H}[\frac{\alpha}{\kappa + i\Delta}(i(1 + \Lambda) + \kappa\Lambda^2)e^{-i\phi}]\rho_{SE}dW, \quad (10)$$

where $\mathcal{L}_{SE}\rho_{SE} = -i[H_{SE}^D, \rho_{SE}] + \kappa D[X]\rho_{SE}$. The relevant detector signal is

$$dQ = 2\eta\kappa\langle \frac{\alpha}{\kappa + i\Delta}(i(1 + \Lambda) + \kappa\Lambda^2)e^{-i\phi} + h.c. \rangle + \sqrt{2\eta\kappa}dW. \quad (11)$$

III. THE MARKOVIAN AND NON-MARKOVIAN NATURE

In order to explore Markovian and non-Markovian nature in spin environments, it is necessary to solve

Eq.(10). Let us first consider a simple two-level system with Hamiltonian $H_S = \Omega_1|1\rangle\langle 1| + \Omega_2|2\rangle\langle 2|$, where $\Omega_2 > \Omega_1$. The system-cavity coupling operator is written by $\hat{\lambda} = \gamma\sigma_x^s$ in Eq.(1), and the system operator $\hat{S} = \sigma_Z^s$ in Eq.(5). And let the system interacts with a classical magnetic field $H_{Sf} = \Omega_f\sigma_z^s$, where the field strength is given by

$$\Omega_f \simeq \Omega_1 - \Omega_2 - \frac{|\alpha|^2\gamma^2}{\Omega_2 - \Omega_1 - \omega_c} - \langle \sum_{k=1}^N g_k \sigma_Z^k \rangle - \frac{1}{2}(\xi_p \alpha^* + \xi_p^* \alpha) \left(\frac{\gamma}{\Omega_2 - \Omega_1 - \omega_c} \right)^2 - |\alpha|^2 \frac{\gamma^2}{\Omega_2 - \Omega_1 - \omega_c}, \quad (12)$$

where the expectation $\langle \sum_{k=1}^N g_k \sigma_Z^k \rangle$ denotes the energy level gap of the system created by the environments. This field can help the photons leaking out of the cavity carry the information of the system, which can be demonstrated by the following spectrum diagrams (see Fig.2-Fig.5). It can eliminate irregular part in spectrogram, forming a regular spectrum which can show the dynamics of open system. In Ref. [11], the authors utilized Rabi oscillations field to reveal the non-Markovian effect. In this article, we use a classical magnetic field to adjust energy level gap of the system for eliminating irregular part in spectrogram. As a result, the regular spectrum can help us to detect the nature of Markovianity besides non-Markovianity.

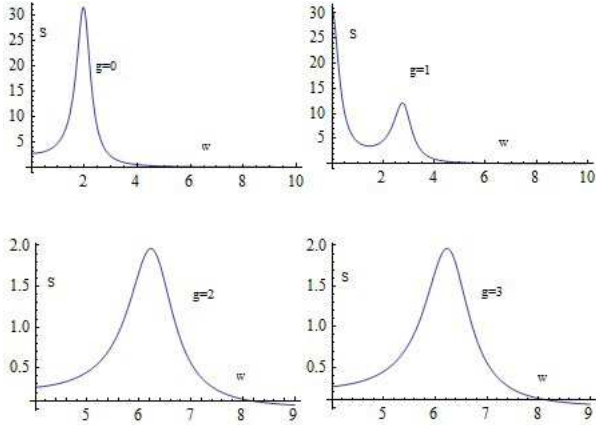


FIG. 2: A qubit coupled to a non-Markovian environment composed of a single spin ($N = 1$). The plot shows the spectrum of the detector current that continuously measures the qubit population. The spectrum S is monotonically shifted and broadened as the coupling strength g increases (representing that non-Markovian effect is strengthened). Here, the corresponding parameters are: $\phi = -\frac{\pi}{2}$, $\Delta = 0$, $\gamma = 2$, $\kappa = 10$, $\xi_p = 10$, $\Omega_2 - \Omega_1 - \omega_c = 20$, $\eta = 1$, and $\alpha = 1$. In this diagram, the spectrum function $S = 1/2S(\omega) - 1$, where the $S(\omega)$ comes from Eq.(14).

Utilizing the $It\hat{o}$ rules: $\langle dW^2 \rangle = dt$, $\langle dW \rangle = 0$, one can obtain a reduced correlation function for the detector current [1]

$$\begin{aligned} R(\tau) &= \langle I(t+\tau)I(t) \rangle - \langle I(t+\tau) \rangle \langle I(t) \rangle \\ &= 2\eta\kappa(\text{Tr}[\hat{x}e^{\mathcal{L}_S\tau}(\hat{c}\rho_S(t) + \rho_S(t)\hat{c}^\dagger)] + \delta(\tau) - \text{Tr}[\hat{x}\rho(t+\tau)]\text{Tr}[\hat{x}\rho(t)]), \end{aligned} \quad (13)$$

where $\hat{c} = \sqrt{2\eta\kappa} \frac{\alpha}{\kappa + i\Delta} (i(1+\Lambda) + \kappa\Lambda^2)e^{-i\phi}$, $\hat{x} = \hat{c} + \hat{c}^\dagger$, and the unconditional master equation $\dot{\rho}_S = e^{\mathcal{L}_S\tau}\rho_S$. The spectrum of the homodyne photon is described by

$$S(\omega) = \lim_{t \rightarrow \infty} \int_{-\infty}^{\infty} d\tau R(\tau) = 2\eta\kappa (1 + \int_{-\infty}^{\infty} d\tau \text{Tr}[\hat{x}e^{\mathcal{L}_S\tau}(\hat{c}\rho_{SS} + \rho_{SS}\hat{c}^\dagger)] - \text{Tr}[\hat{x}\rho_{SS}]^2) \quad (14)$$

where the density matrix ρ_{SS} represents the steady state of the system.

Due to that the Hamiltonian of environment H_E is commutative with the system-environment interaction Hamiltonian H_{CR} , we can get the SME for the system by reducing Eq.(10). And for getting the spectrum, we just need to obtain the steady state of the system. We assume that the initial state of system-environment is of the form $\rho_S^i \otimes \rho_E^i$ and $\rho_E^i = \prod_{j=1}^N [a_j|1\rangle_j\langle 1| + (1-a_j)|2\rangle_j\langle 2|]^{\otimes j}$, where $0 \leq a_j \leq 1$ and $a_j \in \mathcal{R}$. Therefore, we can obtain the unconditional master equation for the system

$$\begin{aligned} \dot{\rho}_S &= \text{Tr}[\mathcal{L}_{SE}\rho_{SE}]_E + \{-i(\xi_p\alpha^* + \alpha^*\xi_p)[\Lambda, \sigma] - \\ &i|\alpha|^2[O_S + \Delta, \rho_S] + \frac{\kappa|\alpha|^2}{\kappa^2 + \Delta^2}D[O_S]\rho_S + \frac{i\Delta|\alpha|^2}{\kappa^2 + \Delta^2}[O_S^2, \rho_S]\}. \end{aligned} \quad (15)$$

The steady state of the system in Eq.(14) can be obtained by solving $\dot{\rho}_S = 0$.

As shown in Fig.2, the peak of spectrum shifts towards the left when we increase the non-Markovian dephasing rate (by increasing the coupling strength g). Here we follow the definition of non-Markovianity and Markovianity in Ref. [17]: non-Markovian effect means that the information of system can come back from environments. The peaks do not shift, and are broader, when increase the Markovian dephasing rate (increase the factor V), as shown in Fig.3, Fig.4 and Fig.5. It signifies that in experiment the spectrum can reveal the nature of a Markovian and non-Markovian dynamics.

IV. CORRELATED ENVIRONMENTS DEMONSTRATED BY WEAK SPECTROSCOPY

Correlated environments have been studied extensively in quantum transport [18], nonlocal non-Markovian dynamics [13, 19], et.al. The authors in Ref. [13] found the nonlocal non-Markovian effect when the local dynamics of subsystem was Markovianity. Therefore, we advise that one can utilize the spectrum from the continuous measurement to detect the nonlocal non-Markovian effect, showing the correlation between environments.

In experiment, one can put two same dimer systems in a cavity, and perform a continuous measurement on leaking photons. The two quantum systems independently interact with the cavity field. So the corresponding interaction operator is described by

$$\hat{\mu} = \hat{\mu}_1 + \hat{\mu}_2. \quad (16)$$

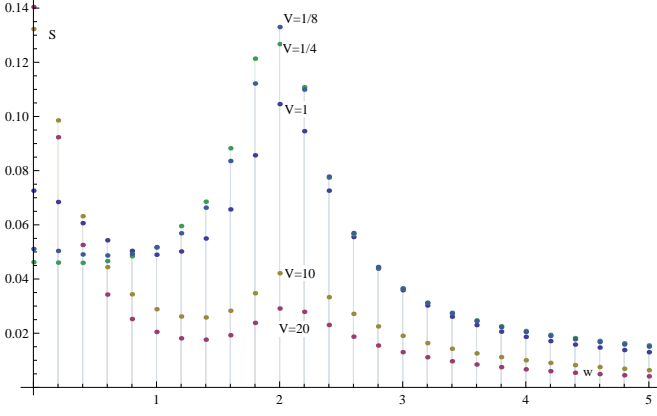


FIG. 3: A quantum system that is coupled to a spin environment is probed by a single mode cavity resonator coupled to a spin environment. By the way of homodyne detection, a detector continuously measures the photons leaking out of the cavity to obtain information about the system dynamics. Here, for the simplicity, we choose the coupling strength $g_k = g$ for $k = 1, 2, \dots, N$. For $N \gg 1$, the state of the environment is $\rho_E^i = \int d\theta \exp[-\sqrt{7}\theta^2/V]|\theta\rangle\langle\theta|$ (meaning that the environment suffers from other extra control or noise), where $\sum_{k=1}^N \sigma_Z^k |\theta\rangle = \theta |\theta\rangle$ with $\theta \in \mathcal{R}$. As a result, the dephasing rate is proportional to $Vt^{3/2}$. The spectrum is monotonically broadened and the peak maxima decreases as the value V which represents the strength of Markovian decoherence. Here, the corresponding parameters are: $\phi = -\frac{\pi}{2}$, $\Delta = 0$, $\gamma = 2$, $\kappa = 10$, $\xi_p = 10$, $\Omega_2 - \Omega_1 - \omega_c = 20$, $\eta = 1$, and $\alpha = 1$. In this diagram, the spectrum function $S = 1/10(\frac{S(\omega)}{2} - 1)$, where the function $S(\omega)$ comes from Eq.(14).

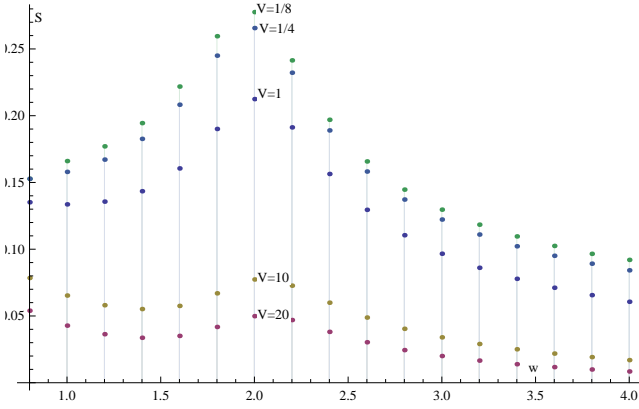


FIG. 4: Similar to Fig.3, in this diagram we consider that the Markovian dephasing rate is proportional to Vt^2 . The state of the corresponding environment is $\rho_E^i = \int d\theta \exp[-\theta^2/V]|\theta\rangle\langle\theta|$.

The detector current operator is $I(t) = I_1(t) + I_2(t)$, where $I_i = \text{Tr}[(c_i + c_i^\dagger) \exp[\mathcal{L}_i](c_i \rho + \rho c_i^\dagger)]$. The reduced correlation function is obtained

$$\begin{aligned} R(\tau) &= \langle I(t+\tau)I(t) \rangle - \langle I(t+\tau) \rangle \langle I(t) \rangle \\ &= R_1(\tau) + R_2(\tau) + R_c(\tau), \end{aligned} \quad (17)$$

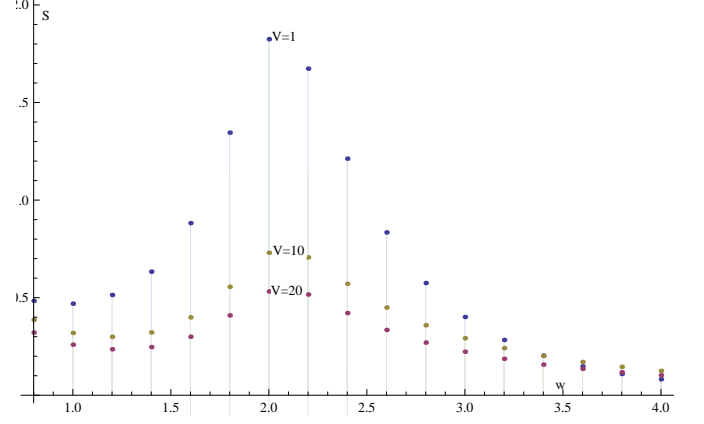


FIG. 5: Like Fig.3 and Fig.4, the Markovian dephasing rate is proportional to Vt . The state of the environment is $\rho_E^i = \int d\theta \exp[-\theta^2 t/V]|\theta\rangle\langle\theta|$.

where the function $R_i = \langle I_i(t+\tau)I_i(t) \rangle - \langle I_i(t+\tau) \rangle \langle I_i(t) \rangle$ for $i = 1, 2$, and the function

$$\begin{aligned} R_c(\tau) &= 2\eta\kappa(\text{Tr}[\hat{x}_1 e^{\mathcal{L}_S \tau}(\hat{c}_2 \rho_S(t) + \rho_S(t) \hat{c}_2^\dagger)] - \delta(\tau) \\ &\quad - \text{Tr}[\hat{x}_1 \rho(t+\tau)] \text{Tr}[\hat{x}_2 \rho(t)] + \text{Tr}[\hat{x}_2 e^{\mathcal{L}_S \tau}(\hat{c}_1 \rho_S(t) \\ &\quad + \rho_S(t) \hat{c}_1^\dagger)] - \text{Tr}[\hat{x}_2 \rho(t+\tau)] \text{Tr}[\hat{x}_1 \rho(t)]), \end{aligned} \quad (18)$$

which reflects the correlation between environments. Hence, one can detect the nonlocal non-Markovian effect by the spectrum shift. This way only helps to detect the correlation between neighbouring environments in a single cavity. It will be significant to further study the way for detecting the correlations between long range environments by the spectrum technique, which is left as an open question.

V. CONCLUSION AND OUTLOOK

We explore continuous measurement in spin environments. By monitoring the photons leaking out of the cavity, the system dynamics can be uncovered. A classical magnetic field play an important role in adjusting the energy level gap of system. Therefore, we obtain that Markovian and non-Markovian dynamics have a different phenomenon in the spectroscopy, which can be used to detect correlated environments.

In this article, we consider some simple spin environments. Hence, more complicated spin environments deserve further study. In another word, it is meaningful to find a universal method to deal with the continuous measurement in spin environments. There are many applications of continuous measurement in quantum information process, such as quantum state reconstruction [20], Bell measurements [21], the quantum Cramér-Rao sensitivity limit [22], correcting low-frequency noise [23], etc. Furthermore, the next research direction is to apply continuous measurement in different fields. In particular, we

are interested in developing continuous measurement in relativistic quantum metrology [24].

No. 11375168.

VI. ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China under Grant No. 10975125 and

-
- [1] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control* (Cambridge University Press, USA, 2010).
 - [2] A. Shabani and K. Jacobs, *Phys. Rev. Lett.* **101**, 230403 (2008).
 - [3] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).
 - [4] D. A. Lidar and T. A. Brun, Ed., *Quantum Error Correction* (Cambridge University Press, Cambridge, UK, 2013).
 - [5] A. Blais, R. S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, *Phys. Rev. A* **69**, 062320 (2004).
 - [6] S. Habib, K. Jacobs, and K. Shizume, *Phys. Rev. Lett.* **96**, 010403 (2006).
 - [7] M. J. Everitt, T. D. Clark, P. B. Stiffell, J. F. Ralph, A. Bulsara, and C. Harland, *New J. Phys.* **7**, 64 (2005).
 - [8] A. C. Doherty and H. Mabuchi, *Atoms in microcavities*, in *Optical microcavities*, K. Vahala, Ed. (World Scientific Press, Singapore, 2004).
 - [9] J. Gambetta, A. Blais, M. Boissonneault, A. A. Houck, D. I. Schuster, and S. M. Girvin, *Phys. Rev. A* **77**, 012112 (2008).
 - [10] M. Boissonneault, J. M. Gambetta, and A. Blais, *Phys. Rev. A* **79**, 013819 (2009).
 - [11] A. Shabani, J. Roden, and K. B. Whaley, *Phys. Rev. Lett.* **112**, 113601 (2014).
 - [12] L. Amico, A. Di Lorenzo, and A. Osterloh, *Phys. Rev. Lett.* **86**, 5759 (2001); M. Bortz and J. Stolze, *Phys. Rev. B* **76**, 014304 (2007); E. Barnes, L. Cywinski, and S. D. Sarma, *Phys. Rev. Lett.* **109**, 140403 (2012).
 - [13] Elsi-Mari Laine, Heinz-Peter Breuer, Jyrki Piilo, Chuan-Feng Li, and Guang-Can Guo, *Phys. Rev. Lett.* **108**, 210402 (2012).
 - [14] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).
 - [15] A. C. Doherty and K. Jacobs, *Phys. Rev. A* **60**, 2700 (1999).
 - [16] C. L. Hutchison, J. M. Gambetta, A. Blais, F. K. Wilhelm, *Can. J. Phys.* **87**, 225 (2009).
 - [17] Heinz-Peter Breuer, Elsi-Mari Laine, and Jyrki Piilo, *Phys. Rev. Lett.* **103**, 210401 (2009).
 - [18] I. Sinayskiy, A. Marais, F. Petruccione, and A. Ekert, *Phys. Rev. Lett.* **108**, 020602 (2012); Mohan Sarovar, Yuan-Chung Cheng, and K. B. Whaley, *Phys. Rev. E* **83**, 011906 (2011).
 - [19] Salvatore Lorenzo, Francesco Plastina, and Mauro Paternostro, *Phys. Rev. A* **84**, 032124 (2011); M.B. Hastings, I. Martin, and D. Mozyrsky, *Phys. Rev. B* **68**, 035101 (2003); Dong Xie and An Min Wang, *Chin. Phys. B* **23**, 040302 (2014).
 - [20] Andrew Silberfarb, Poul S. Jessen, and Ivan H. Deutsch, *Phys. Rev. Lett.* **95**, 030402 (2005).
 - [21] Sebastian G. Hofer, Denis V. Vasilyev, Markus Aspelmeyer, and Klemens Hammerer, *Phys. Rev. Lett.* **111**, 170404 (2013).
 - [22] Søren Gammelmark and Klaus Mølmer, *Phys. Rev. Lett.* **112**, 170401 (2014).
 - [23] L. Tian, *Phys. Rev. Lett.* **98**, 153602 (2007).
 - [24] Mehdi Ahmadi, David Edward Bruschi, Carlos Sabín, Gerardo Adesso, and Ivette Fuentes, *Sci. Rep.* **4**, 4996 (2014).